ECEN Problem 7.2

For $f = 1\,\text{GHz}$, $\mu_r = 1$, and $\varepsilon_r = 9$,

$\omega = 2\pi f = 2\pi \times 10^9$ rad/s,

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_r} = \frac{2\pi f}{c}\sqrt{\varepsilon_r} = \frac{2\pi \times 10^9}{3\times 10^8}\sqrt{9} = 20\pi \text{ rad/m}$$

$$E(y,t) = \hat{x}\, 6\cos(2\pi \times 10^9 t - 20\pi y + \phi_0)\ (\text{V/m})$$

At $t=0$ and $y = 2$ cm, $E = 4$ V/m :

$$4 = 6\cos(-20\pi \times 2\times 10^{-2} + \phi_0) = 6\cos(-0.4\pi + \phi_0)$$

Hence, $\phi_0 - 0.4\pi = \cos^{-1}\left(\frac{4}{6}\right) = 0.84$ rad

which gives $\phi_0 = 2.1$ rad $= 120.19°$

and $\boxed{E(y,t) = \hat{x}\, 6\cos(2\pi \times 10^9 t - 20\pi y + 120.19°)\ (\text{V/m})}$

ECEN Problem 7.4

(a) Since $k = 0.2\pi$,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2\pi} = \boxed{10\text{m}}$$

(b) $$u_p = \frac{\omega}{k} = \pi \times 10^7 / 0.2\pi = 5 \times 10^7 \text{ m/s}$$

But $$u_p = c/\sqrt{\varepsilon_r}$$

Hence, $$\varepsilon_r = \left(\frac{c}{u_p}\right)^2 = \left(\frac{3\times10^8}{5\times10^7}\right)^2 = \boxed{36}$$

(c) $$H = \frac{1}{\eta}\hat{k}\times E = \frac{1}{\eta}\hat{x}\times[\hat{y}3\sin(\pi\times10^7t - 0.2\pi x) + \hat{z}4\cos(\pi\times10^7t - 0.2\pi x)]$$

$$= \hat{z}\frac{3}{\eta}\sin(\pi\times10^7t - 0.2\pi x) - \hat{y}\frac{4}{\eta}\cos(\pi\times10^7t - 0.2\pi x)\ (\text{A/m})$$

With $\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} \simeq \frac{120\pi}{6} = 20\pi = 62.83\ (\Omega)$

ECEN Problem 7.6

(a) $\omega = 2\pi f = 6\pi \times 10^9$ rad/s

$f = 3\times 10^9$ HZ $= 3$ GHZ

$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times 10^8}{\sqrt{2.56}} = 1.875\times 10^8$ m/s

$\lambda = u_p/f = \frac{1.875\times 10^8}{\cancel{3}\times 10^9} = \cancel{3.12\,cm}\ 6.3$ cm

$k = \cancel{\frac{2\pi}{\lambda}} = \frac{2\pi}{\cancel{3.12\times 10^{-2}}\ 6.3\times 10^{-2}} = \cancel{201.4\ rad/m}\ 100.3\ \cancel{rad}/m$

$\eta = \eta_0/\sqrt{\varepsilon_r} = \frac{377}{\sqrt{2.56}} = \frac{377}{1.6} = 235.62\,\Omega$

(b) $H = -\hat{x}\frac{20}{\eta}\cos(6\pi\times 10^9 t - kz)$

$= -\hat{x}\frac{20}{235.62}\cos(6\pi\times 10^9 t - \cancel{201.4}\ 100.3\, z)$

$= -\hat{x}\, 8.49\times 10^{-2}\cos(6\pi\times 10^9 t - \cancel{201.4z}\ 100.3z)$ (A/m)

ECEN Problem 7.10

$\tilde{E} = \hat{x} a_x e^{-jkz}$

RHC wave : $\tilde{E}_R = a_R(\hat{x} + \hat{y} e^{-j\pi/2}) e^{-jkz} = a_R(\hat{x} - j\hat{y}) e^{-jkz}$

LHC wave : $\tilde{E}_L = a_L(\hat{x} + \hat{y} e^{j\pi/2}) e^{-jkz} = a_L(\hat{x} + j\hat{y}) e^{-jkz}$

$\tilde{E} = \tilde{E}_R + \tilde{E}_L$

$\hat{x} a_x = a_R(\hat{x} - j\hat{y}) + a_L(\hat{x} + j\hat{y})$

By equation real and imaginary parts,

$a_x = a_R + a_L$, $0 = -a_R + a_L$, or $a_L = a_x/2$ $a_R = a_x/2$

ECEN 7.12

(a) $E_1 = \hat{x}\, 2\cos(\omega t - kz) + \hat{y}\, 2\sin(\omega t - kz)$

$= \hat{x}\, 2\cos(\omega t - kz) + \hat{y}\, 2\cos(\omega t - kz - \pi/2)$

$\tilde{E}_1 = \hat{x}\, 2e^{-jkz} + \hat{y}\, 2e^{-jkz}\, e^{-j\pi/2}$

$\psi_0 = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1} 1 = 45°$

$\delta = -\pi/2$ Hence, $\boxed{\text{wave 1 is RHC}}$

$\tilde{E}_2 = \hat{x}\, 2e^{jkz} + \hat{y}\, 2e^{jkz} e^{-j\pi/2}$

Wave 2 has the same magnitude and phases as wave 1 except that its direction is along $-\hat{z}$ instead of $+\hat{z}$. Hence, the locus of rotation of E will match the left hand instead of the right hand. Thus, $\boxed{\text{wave 2 is LHC.}}$

(b) $E_1 = \hat{x}\, 2\cos(\omega t - kz) - \hat{y}\, 2\sin(\omega t - kz)$

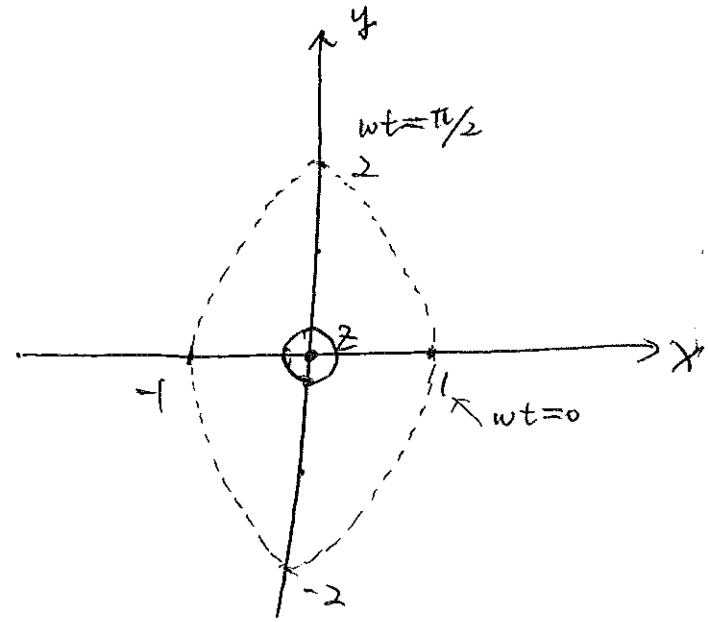
$\tilde{E}_1 = \hat{x}\, 2\, e^{-jkz} + \hat{y}\, 2\, e^{-jkz} e^{j\pi/2}$

$\boxed{\text{Wave 1 is LHC.}}$

$\tilde{E}_2 = \hat{x}\, 2e^{jkz} + \hat{y}\, 2e^{jkz} e^{j\pi/2}$

Reversal of direction of propagation (relative to wave 1) makes $\boxed{\text{wave 2 RHC}}$

ECEN Problem 7.13



$E = \hat{x} \sin(\omega t + kz) + \hat{y}\, 2\cos(\omega t + kz)$

Wave direction is $-\hat{z}$. At $z = 0$,

$$E = \hat{x} \sin\omega t + \hat{y}\, 2\cos\omega t$$

Tip of E rotates in accordance with right hand (with thumb pointing along $-\hat{z}$).

Hence, wave state is RHEC

ECEN Problem 7.16

The phase angle by which the magnetic field leads the electric field is $-\theta_\eta$ where θ_η is the phase angle of η_c

$$\frac{\sigma}{\omega\varepsilon} = \frac{0.1 \times 36\pi}{2\pi \times 10^{8} \times 10^{-9} \times 9} = 2$$

Hence, quasi-conductor

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon'}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-\frac{1}{2}}$$

$$= \frac{120\pi}{\sqrt{\varepsilon_r}}\left(1 - j\frac{\sigma}{\omega\varepsilon_0\varepsilon_r}\right)^{-1/2}$$

$$= 125.67\,(1 - j2)^{-\frac{1}{2}}$$

$$= 71.49 + j\,44.18$$

$$= 84.04\boxed{\angle 31.72^\circ}$$

ECEN Problem 7.18

$$E(z) = E_0 e^{-\alpha z} = 10 e^{-\alpha z}$$

$$\frac{\sigma}{\omega \varepsilon} = \frac{5 \times 10^{-4} \times 36\pi}{2\pi \times 3 \times 10^9 \times 10^{-9} \times 9} = 3.32 \times 10^{-4}$$

Hence, medium is a low-loss dielectric

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\sigma}{2} \cdot \frac{120\pi}{\sqrt{\varepsilon_r}} = \frac{5 \times 10^{-4} \times 120\pi}{2 \times \sqrt{9}} = 0.032 \text{ (Np/m)}$$

$$10^{-3} = 10 e^{-0.032z}, \quad \ln 10^{-4} = -0.032z$$

$$\boxed{z = 287.82\text{ m}}$$